Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Advanced Calculus

Subject Code : 4SC03MTC1 **Branch: B.Sc.(Mathematics)**

Semester : 3 **Date : 23/03/2017** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Find asymptotes of the curve $x^2y^2 = a^2(x^2+y^2)$. parallel to co ordinate axis.11 **(02)**

b) Find interval on which the function $x^3 - x - 5$ is increasing or decreasing. **(02)**

c) Verify Euler's theorem for $u = x^2 + 2axy + y^2$. **(02)**

d) If $x = r\cos\theta, y = r\sin\theta$ then what is the value of $\frac{\partial(x,y)}{\partial(r,\theta)}$? **(02)**

e) If $x^4 + y^4 = 4b^2xy$, find $\frac{dy}{dx}$. **(02)**

f) Prove that $\beta(m,n) = \beta(m,n+1) + \beta(m+1,n)$. **(02)**

g) Is the function $f(x,y) = \sin(\frac{x-y}{x+y})$ homogeneous? **(01)**

h) Write the relation between Beta and Gama function. **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) State and prove Taylor's theorem for the function of two variables. **(08)**

b) Find extreme value of $f(x,y) = x^2 + 2y^2 - x$. **(06)**

Q-3 **Attempt all questions** **(14)**

a) State and prove Duplication formula. **(08)**

b) Evaluate: $\int_0^1 x^m \left(\log\frac{1}{x}\right)^n\ dx$. **(06)**

Q-4 **Attempt all questions** **(14)**

a) If $u = x+y+z, uv = y+z, uvw = z$ then prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)}\,\frac{\partial(x,y,z)}{\partial(u,v,w)} = 1$. **(08)**

b) If $u = \log(\tan x + \tan y + \tan z)$ then prove that: **(06)**
$\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 2$.

Q-5 **Attempt all questions** **(14)**

a) Using definition of limit prove that $\lim_{(x,y)\to(2,4)} x^2 + 5y = 24$. **(05)**

b) a) If u is a homogeneous function of degree n then prove that **(05)**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x\, \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n\,(n-1)\,u.$$

c) If $u(x, y) = \log(\frac{x^2+y^2}{xy})$ then check whether $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ holds. **(04)**

Q-6 **Attempt all questions** **(14)**

a) State and prove Euler's theorem for homogeneous function of two variables. **(05)**

b) Prove that $y = x + 2$ is an asymptote of the curve $y = \frac{x^2+2x-1}{x}$. **(05)**

c) If $u = \tan^{-1}(\frac{y^2}{x})$ find value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x\, \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$. **(04)**

Q-7 **Attempt all questions** **(14)**

a) Expand $e^x \sin y$ in powers of x and y up to three degree. **(05)**

b) Find the maximum value of $f(x, y, z) = xyz$ subject to the constraint $2x + 2y + z = 108$ using Lagrange's method of undetermined multipliers **(05)**

c) Prove that $\Gamma n = \int_0^1 (\log\frac{1}{x})^{n-1} dx$. **(04)**

Q-8 **Attempt all questions** **(14)**

a) Find range of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upward and downward. Also find points of inflection in each case **(05)**

b) Find all asymptotes of the curve $x^3 + y^3 - 3axy = 0$. **(05)**

c) Evaluate: $\int_0^1 \frac{1}{\sqrt{1-x^6}}\, dx$. **(04)**